Imprint of modified Einstein's gravity on white dwarfs: Unifying type Ia supernovae

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Abstract

We establish the importance of modified Einstein's gravity (MG) in white dwarfs (WDs) for the first time in the literature. We show that MG leads to significantly sub- and super-Chandrasekhar limiting mass WDs, depending on a single model parameter. However, conventional WDs on approaching Chandrasekhar's limit are expected to trigger type Ia supernovae (SNeIa), a key to unravel the evolutionary history of the universe. Nevertheless, observations of several peculiar, under- and over-luminous SNeIa argue for the limiting mass widely different from Chandrasekhar's limit. Explosions of MG induced sub- and super-Chandrasekhar limiting mass WDs explain under- and over-luminous SNeIa respectively, thus unifying these two apparently disjoint sub-classes. Our discovery questions both the global validity of Einstein's gravity and the uniqueness of Chandrasekhar's limit.

Introduction

The validity of Einstein's theory of general relativity (GR) has been tested extensively in the weak field regime, e.g., through laboratory experiments and solar-system tests. The question is, whether GR is the ultimate theory of gravitation, or it requires modification in the strong gravity regime. Indeed, it was shown that modified gravity (MG) theories reveal significant deviations to the GR solutions for neutron stars (NSs) [1]. As NSs are much more compact than white dwarfs (WDs), so far, MG theories have been applied only to them in order to test the validity of such theories in the strong field regime. The current venture is to show that a MG theory is indispensable in the context of WDs, which is a first in the literature to the best of our knowledge. The motivation is the following.

Type Ia supernovae (SNeIa) are believed to result from the violent thermonuclear explosion of a carbon-oxygen WD, when its mass approaches the famous Chandrasekhar limit of $1.44M_\odot$, when $M_\odot$ is the solar mass. SNIa is used as a standard candle in understanding the expansion history of the universe [2].

However, some of these SNeIa are highly over-luminous, e.g. SN 2003fg, SN 2006gz, SN 2007if, SN 2009dc [3, 4], and some others are highly under-luminous, e.g. SN 1991bg, SN 1997cn, SN 1998de, SN 1999by [5, 6]. The luminosity of the former group (super-SNeIa) implies highly super-Chandrasekhar WDs, having mass $2.1-2.8M_\odot$, as their most plausible progenitors [3, 4]. While, the latter group (sub-SNeIa) predicts the progenitor mass could be as low as $\sim M_\odot$ [5]. Moreover, the characteristic lightcurves of these SNeIa are quite peculiar compared to their conventional counterparts. The models attempted to explain them so far entail caveats.

A major concern, however, is that a large array of models is required to explain apparently the same phenomena, i.e., triggering of thermonuclear explosions in WDs. It is unlikely that nature would seek mutually antagonistic scenarios to exhibit sub- and super-SNeIa. This is where the current work steps in, which unifies the phenomenologically disjoint sub-classes of SNeIa by a single underlying theory. This is achieved by invoking a MG theory in WDs.

Basic equations and modified gravity model

Let us start with the 4-dimensional action as [7]

$$S = \int \left[\frac{1}{16\pi} f(R) + \mathcal{L}_M\right] \sqrt{-g}\ d^4x, \tag{1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, $\mathcal{L}_M$ the Lagrangian density of the matter field, R the scalar curvature defined as $R = g^{\mu\nu} R_{\mu\nu}$, when $R_{\mu\nu}$ is the Ricci tensor and f is an arbitrary function of R (in GR $f(R) = R$). For the present purpose, we choose the Starobinsky model [8] defined as $f(R) = R + \alpha R^2$, when α is a constant. However, similar effects could also be obtained in other MG theories, e.g. Born-Infeld gravity (e.g. [9]). Now, on extremizing the above action one obtains the modified field equation as

$$G_{\mu\nu} + \alpha X_{\mu\nu} = 8\pi T_{\mu\nu}, \tag{2}$$

where $G_{\mu\nu}$ is Einstein's field tensor, $T_{\mu\nu}$ is the energy-momentum tensor of the matter field and $X_{\mu\nu}$ is a function of $g_{\mu\nu}$, $R_{\mu\nu}$ and R.

Solution procedure

In order to solve equation (2), we adopt the perturbative method (e.g. [10]), such that $\alpha R << 1$. Further, we consider the hydrostatic equilibrium condition: $g_{\nu r}\nabla_\mu T^{\mu\nu} = 0$, with zero velocity and ∇_μ the covariant derivative. Hence, we obtain the differential equations for mass $M_\alpha(r)$, pressure $P_\alpha(r)$ (or density $\rho_\alpha(r)$) and gravitational potential $\phi_\alpha(r)$, of spherically symmetric WDs, where r is the radial coordinate: the *modified* Tolman-Oppenheimer-Volkoff (TOV) equations. For $\alpha = 0$, these equations reduce to TOV equations in GR.

In order to solve the modified TOV equations, we must supplement them with appropriate boundary conditions and an equation of state (EoS) obtained by Chandrasekhar [11], given by $P_\alpha = K\rho_\alpha^{1+(1/n)}$, for extremely low and high densities, where n is the polytropic index and K a dimensional constant. The boundary conditions are $M_\alpha(0) = 0$ and $\rho_\alpha(0) = \rho_c$, where ρ_c is the central density of the WD. Note that a particular ρ_c corresponds to a particular mass M_* and radius R_* of WDs. Hence, by varying ρ_c from 2×10^5 gm/cc to 10^{11} gm/cc, we construct the mass-radius relation.

Results

Figures 1(a) and (b) show that, for $\alpha = 0$ (GR case), with the increase of ρ_c, M_* increases and R_* decreases, until it reaches a maximum mass $M_{\max} = 1.405M_\odot$ (smaller than the Newtonian Chandrasekhar limit of $1.44M_\odot$) at $\rho_c = 3.5 \times 10^{10}$ gm/cc. A further increase in ρ_c results in a slight decrease in M_*, indicating the onset of an unstable branch.

Coming to the $\alpha < 0$ cases, Figure 1(b) shows that for $\rho_c > 10^8$ gm/cc, the $M_* - \rho_c$ curves deviate from the GR curve due to MG effects. This reveals that MG has a tremendous impact on WDs which so far was completely overlooked. Note that $M_{\max}$ for all the three cases corresponds to $\rho_c = 10^{11}$ gm/cc, an upper-limit chosen to avoid possible neutronization. Interestingly, all values of $M_{\max}$ are highly super-Chandrasekhar, ranging from $1.8 - 2.7M_\odot$. Thus while the GR effect is very small, MG effect could lead to $\sim 100\%$ increase in the limiting mass of WDs. The corresponding values of ρ_c are large enough to initiate thermonuclear reactions, e.g. they are larger than ρ_c corresponding to $M_{\max}$ of $\alpha = 0$ case, whereas the respective core temperatures are expected to be similar. This explains the entire range of the observed super-SNeIa mentioned above [3, 4].

Table 1 ensures the perturbative validity of the solutions. Recall that we solve the modified TOV equations only up to $\mathcal{O}(\alpha)$. Since the product αR is first order in α, we replace R in it by the zeroth order Ricci scalar $R^{(0)} = 8\pi(\rho^{(0)} - 3P^{(0)})$, where $\rho^{(0)}$ and $P^{(0)}$ are the zeroth order solutions of density and pressure respectively, obtained in GR (when $\alpha = 0$). For the perturbative validity of the entire solution, $|\alpha R^{(0)}|_{\max} \ll 1$ should hold true. Next we consider $g_{tt}^{(0)}/g_{tt}$ and $g_{rr}^{(0)}/g_{rr}$ (ratios of $g_{\mu\nu}$-s in GR and those in MG up to $\mathcal{O}(\alpha)$), which should be close to 1 for the validity of perturbative method [13]. Hence, $|1 - g_{tt}^{(0)}/g_{tt}|_{\max} \ll 1$ and $|1 - g_{rr}^{(0)}/g_{rr}|_{\max} \ll 1$ should both hold true. Table 1 shows that all the three measures quantifying perturbative validity are at least $2 - 3$ orders of magnitude smaller than 1.

Coming to the $\alpha > 0$ cases, Figure 1(b) shows that all the three $M_* - \rho_c$ curves overlap with the $\alpha = 0$ curve in the low density region. However, with the increase of α, the region of overlap recedes to a lower ρ_c. MG effects set in at $\rho_c \gtrsim 10^8,\ 4 \times 10^7$ and 2×10^6 gm/cc,

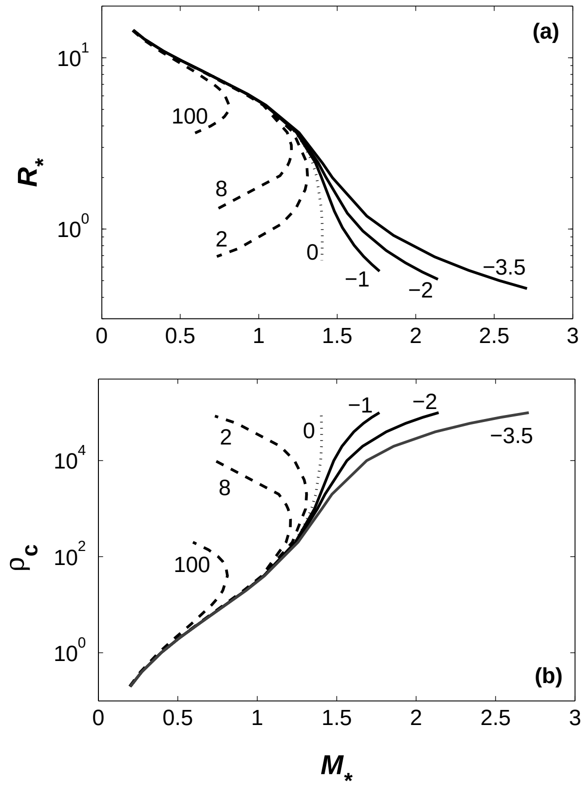

Figure 1: Unification diagram for SNeIa: (a) Mass-radius relations. (b) Variation of ρ_c with M_*. The numbers adjacent to the various lines denote $\alpha/(10^{13}\ \mathrm{cm}^2)$. ρ_c, M_* and R_* are in units of 10^6 gm/cc, $M_\odot$ and 1000 km, respectively.

for $\alpha = 2 \times 10^{13}$ cm^2, 8×10^{13} cm^2 and 10^{15} cm^2 respectively. For a given α, with the increase of ρ_c, M_* first increases, reaches a maximum and then decreases, like the $\alpha = 0$ case. With the increase of α, $M_{\rm max}$ decreases and, interestingly, for $\alpha = 10^{15}$ cm^2, it is highly sub-Chandrasekhar ($0.81 M_\odot$). In fact, $M_{\rm max}$ for all the chosen $\alpha > 0$ is sub-Chandrasekhar, ranging $1.31 - 0.81 M_\odot$. This is a remarkable finding since it establishes that even if ρ_c-s for these sub-Chandrasekhar WDs are lower than the conventional value at which SNeIa are usually triggered, an attempt to increase the mass beyond $M_{\rm max}$ with increasing ρ_c, for a given α, will lead to a gravitational instability. This presumably will be followed by a runaway thermonuclear reaction, provided the core temperature increases sufficiently due to collapse. Occurrence of such thermonuclear runway reactions, triggered at densities as low as 10^6 gm/cc, has already been demonstrated [14]. Thus, once $M_{\rm max}$ is approached, a SNIa is expected to trigger just like in the $\alpha = 0$ case, explaining the sub-SNeIa [5, 6], like SN 1991bg mentioned above. Table 2 confirms that the solutions for the $\alpha > 0$ cases are within the perturbative regime.

Table 1: Measure of validity of perturbative solutions for $\alpha < 0$ corresponding to $M_{\rm max}$ in Fig. 1.

$\alpha/(10^{13}\ \mathrm{cm}^2)$	$\lvert\alpha R^{(0)}\rvert_{\rm max}$	$\lvert 1-g_{tt}^{(0)}/g_{tt}\rvert_{\rm max}$	$\lvert 1-g_{rr}^{(0)}/g_{rr}\rvert_{\rm max}$
-1	0.00184	0.0016	0.0052
-2	0.00369	0.0031	0.0108
-3.5	0.00646	0.0052	0.0199

Table 2: Measure of validity of perturbative solutions for $\alpha > 0$ corresponding to $M_{\rm max}$ in Fig. 1.

$\alpha/(10^{13}\ \mathrm{cm}^2)$	$\lvert\alpha R^{(0)}\rvert_{\rm max}$	$\lvert 1-g_{tt}^{(0)}/g_{tt}\rvert_{\rm max}$	$\lvert 1-g_{rr}^{(0)}/g_{rr}\rvert_{\rm max}$
2	7.4×10^{-5}	6.8×10^{-5}	2.0×10^{-4}
8	7.4×10^{-5}	6.8×10^{-5}	2.0×10^{-4}
100	7.4×10^{-5}	6.9×10^{-5}	2.0×10^{-4}

Conclusions

Based on a simple $f(R)$-model, we show that modifications to GR are indispensable in WDs, especially for determining their limiting mass. It remarkably explains and unifies a wide range of observations for which GR is insufficient. We note here that the perturbative method is adequate for the present study, as then we have a handle on α characterizing our model which cannot be arbitrarily large, allowing it to be constrained directly by astrophysical observations [15]. Hence, depending on the magnitude and sign of α, we not only obtain both highly super-Chandrasekhar (for $\alpha < 0$) and highly sub-Chandrasekhar (for $\alpha > 0$) limiting mass WDs, but we also establish them as progenitors of the peculiar, over-luminous and under-luminous SNeIa, respectively. We further note that even though α is assumed to be constant within individual WDs, there is indeed an implicit dependence of α on ρ_c, as evident from Figure 1(b), indicating the existence of an underlying chameleon effect [16]. Thus, a single underlying theory, inspired by the need to modify Einstein's theory of GR, unifies the two apparently disjoint sub-classes of SNeIa, which have so far hugely puzzled astronomers. The significance of the current work lies in the fact that it not only questions the uniqueness of the Chandrasekhar mass-limit for WDs, but it also argues for the need of a *modified theory* of GR to explain the observable universe.

References

[1] T. Damour and G. Esposito-Farése, Phys. Rev. Lett. **70,** 2220 (1993).

[2] S. Perlmutter *et al.*, Astrophys. J. **517,** 565 (1999).

[3] D.A. Howell *et al.*, Nature **443,** 308 (2006).

[4] R.A. Scalzo *et al.*, Astrophys. J. **713,** 1073 (2010).

[5] A.V. Filippenko *et al.*, Astron. J. **104,** 1543 (1992).

[6] S. Taubenberger *et al.*, Mon. Not. R. Astron. Soc. **385,** 75 (2008).

[7] A. de Felice and S. Tsujikawa, Liv. Revs in Rel. **13,** 3 (2010).

[8] A.A. Starobinsky, Phys. Lett. B **91,** 99 (1980).

[9] M. Banados and P.G. Ferreira, Phys. Rev. Lett. **105,** 011101 (2010).

[10] A.V. Astashenok, S. Capozziello and S.D. Odintsov, JCAP **12,** 040 (2013).

[11] S. Chandrasekhar, Mon. Not. R. Astron. Soc. **95,** 207 (1935).

[12] R.F. Tooper, Astrophys. J. **140,** 434 (1964).

[13] M. Orellana, F. García, F.A. Teppa Pannia and G.E. Romero, Gen. Rel. Grav. **45,** 771 (2013).

[14] I.R. Seitenzahl, C.A. Meakin, D.M. Townsley, D.Q. Lamb and J.W. Truran, Astrophys. J. **696,** 515 (2009).

[15] J. Näf and P. Jetzer, Phys. Rev. D **81,** 104003 (2010).

[16] T. Faulkner, M. Tegmark, E.F. Bunn, Y. Mao, Phys. Rev. D **76,** 063505 (2007).